Date: 18/05/2025

Time : 3 hrs.

Answers & Solutions

for

JEE (Advanced)-2025 (Paper-1)

Max. Marks: 180

PART-I : MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

1. Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$.

Define the functions $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$, and $h : \mathbb{R} \to \mathbb{R}$ by

$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4$,

$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4$,

$h(x) = f(x + 1) - g(x + 2)$.

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of x^3 in $h(x)$ is

(A) 8 (B) 2

(C) –4 (D) –6

Answer (C)

Sol. $f(x+1)=a_1+10(x+1)+a_2(x+1)^2+a_3(x+1)^3+(x+1)^4$

Coefficient of x^3 in $f(x+1) = a_3 + 4$

$g(x+2)=b_1+3(x+2)+b_2(x+2)^2+b_3(x+2)^3+(x+2)^4$

Coefficient of x^3 in $g(x+2) = b_3 + 8$

$\Rightarrow$ Coefficient of x^3 in $h(x) = f(x+1) - g(x+2)$

is $a_3 + 4 - b_3 - 8 = a_3 - b_3 - 4$

But $f(x) \neq g(x)\ \forall\ x$

$\Rightarrow$ $f(x) - g(x) \neq 0$

$\Rightarrow$ $f(x) - g(x) = 0$ have no real roots

$(a_1 - b_1) + 7x + (a_2 - b_2)x^2 + (a_3 - b_3)x^3 = 0$ have no real roots

$\Rightarrow$ $a_3 - b_3 = 0$

$\Rightarrow$ Coefficient of x^3 in $h(x) = -4$

Option (C) is correct.

2. Three students S_1, S_2, and S_3 are given a problem to solve. Consider the following events:

U: At least one of S_1, S_2, and S_3 can solve the problem,

V: S_1, can solve the problem, given that neither S_2 nor S_3 can solve the problem,

W: S_2 can solve the problem and S_3 cannot solve the problem,

T: S_3 can solve the problem.

For any event E, let $P(E)$ denote the probability of E. If

$$P(U)=\frac{1}{2},\ P(V)=\frac{1}{10},\ \text{and}\ P(W)=\frac{1}{12},$$

then $P(T)$ is equal to

(A) $\frac{13}{36}$

(B) $\frac{1}{3}$

(C) $\frac{19}{60}$

(D) $\frac{1}{4}$

Answer (A)

Sol. Let $\left.\begin{matrix}P(S_1)=a\\P(S_2)=b\\P(S_3)=c\end{matrix}\right\}$ they are independently solving

$$U=S_1\cup S_2\cup S_3 \Rightarrow U^C=\left(\overline{S_1}\cap\overline{S_2}\cap\overline{S_3}\right)$$

$\Rightarrow$ $P(U) = 1 - P(U^C) = 1 - [(1-a)(1-b)(1-c)] = \frac{1}{2}$

$\Rightarrow$ $(1-a)(1-b)(1-c) = \frac{1}{2}$

$$P(V) = P\left(\frac{S_1}{\overline{S_2} \cap \overline{S_3}}\right) = \frac{P\left(S_1 \cap \overline{S_2} \cap \overline{S_3}\right)}{P\left(\overline{S_2} \cap \overline{S_3}\right)} = P(s_1) = a = \frac{1}{10}$$

$$P(W) = P\left(S_2 \cap \overline{S_3}\right) = b(1-c) = \frac{1}{12}$$

$$\left(1 - \frac{1}{10}\right)(1-b)(1-c) = \frac{1}{2} \Rightarrow (1-b)(1-c) = \frac{5}{9}$$

$$\frac{b}{1-b} = \frac{1}{12} \times \frac{9}{5} = \frac{3}{20} \Rightarrow b = \frac{3}{23}$$

$$\frac{3}{23}(1-c) = \frac{1}{12} \Rightarrow 1 - c = \frac{23}{36}$$

$\Rightarrow$ $c = \frac{13}{36}$

$P(T) = P(S_3) = c = \frac{13}{36}$

3. Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\frac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then which one of the following statements is TRUE?

(A) The function f is **NOT** differentiable at $x = 0$

(B) There is a positive real number δ, such that f is a decreasing function on the interval $(0, \delta)$

(C) For any positive real number δ, the function f is **NOT** an increasing function on the interval $(-\delta, 0)$

(D) $x = 0$ is a point of local minima of f

Answer (B)

Sol. $f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\frac{1}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}$

$\lim_{x \to 0} f(x) = 2 = f(0)$ $\therefore$ $f(x)$ is continuous

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$\lim_{h \to 0} \frac{2 - 2h^2 - h^2 \sin\left(\frac{1}{h}\right) - 2}{h}$$

$$\lim_{h \to 0} \frac{-2h - h\sin\left(\frac{1}{h}\right)}{1} = 0$$

$\therefore$ $f'(0) = 0$

$\therefore$ $f(x)$ is differentiable at $x = 0$

$$\text{RHD} = \lim_{h \to 0^+} -h\left(2 + \sin\left(\frac{1}{h}\right)\right) < 0$$

$$\text{LHD} = \lim_{h \to 0^-} -h\left(2 + \sin\left(\frac{1}{h}\right)\right) > 0$$

$\therefore$ $x = 0$ is a point of maxima

$\therefore$ In $(0, \delta)$ function is decreasing $(\delta > 0)$

Option (B) is correct

In $(-\delta, 0)$ function will be increasing $(\delta > 0)$

Option (C) is incorrect

4. Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Let the transpose of a matrix X be denoted by X^T. Then the number of 3×3 invertible matrices Q with integer entries, such that $Q^{-1} = Q^T$ and $PQ = QP$,

(A) 32   (B) 8

(C) 16   (D) 24

Answer (C)

Sol. As $PQ = QP$

$$\therefore \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 3g & 3h & 3i \end{bmatrix} = \begin{bmatrix} 2a & 2b & 3c \\ 2d & 2e & 3f \\ 2g & 2h & 3i \end{bmatrix}$$

$\Rightarrow\ c = 0, f = 0, h = 0, g = 0$

We get $Q = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{bmatrix}$

Given $QQ^T = I$

$$\Rightarrow \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & i \end{bmatrix}\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + b^2 & ac + bd & 0 \\ ac + bd & c^2 + d^2 & 0 \\ 0 & 0 & i^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow\ a^2 + b^2 = 1, ac + bd = 0, c^2 + d^2 = 1, i^2 = 1$

Case I: $(a, b) = (0, 1)$ or $(0, -1)$, $(c, d) = (1, 0)$ or $(-1, 0)$

Case II: $(a, b) = (1, 0)$ or $(-1, 0)$, $(c, d) = (0, 1)$ or $(0, -1)$

Therefore total possible $Q = 16$

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

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5. Let L_1 be the line of intersection of the planes given by the equations

$2x + 3y + z = 4$ and $x + 2y + z = 5$.

Let L_2 be the line passing through the point $P(2, -1, 3)$ and parallel to L_1. Let M denote the plane given by the equation

$2x + y - 2z = 6$

Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to the plane M.

Then which of the following statements is (are) TRUE?

(A) The length of the line segment PQ is $9\sqrt{3}$

(B) The length of the line segment QR is 15

(C) The area of ΔPQR is $\frac{3}{2}\sqrt{234}$

(D) The acute angle between the line segments PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Answer (A, C)

Sol. $L_1 : 2x + 3y + z = 4$

$x + 2y + z = 5$

$\therefore$ line L_1 in standard form is

$$\frac{x+7}{1} = \frac{y-6}{-1} = \frac{z}{1}$$

and equation of line L_2 is:

$$\frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-3}{1}$$

Equation of plane $M : 2x + y - 2z = 6$.

Let coordinate of $Q = (\lambda + 2, -\lambda - 1, \lambda + 3)$.

$\because$ Q lies on plane M

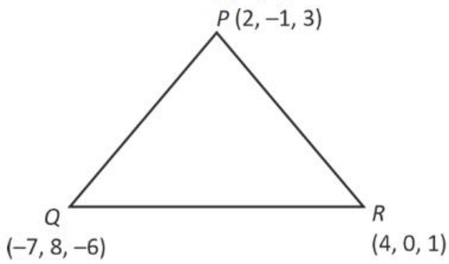
$\therefore \lambda = -9$

$\therefore$ coordinate of $Q = (-7, 8, -6)$.

For foot of perpendicular $R(x_1, y_1, z_1)$

$$\frac{x_1-2}{2} = \frac{y_1+1}{1} = \frac{z_1-3}{-2} = \frac{-(4-1-6-6)}{9}$$

$\therefore$ Coordinate of $R = (4, 0, 1)$



$\therefore PQ = 9\sqrt{3}$ units

$QR = \sqrt{234}$ units

$PR = 3$ units

Let θ be acute angle between PQ and PR, then

$$\cos\theta = \frac{1}{3\sqrt{3}} \text{ and } \sin\theta = \sqrt{\frac{26}{27}}$$

$$\text{Area of } \Delta PQR = \frac{1}{2}. PQ \cdot PR \cdot \sin\theta = \frac{3}{2}\sqrt{234} \text{ sq. units}$$

6. Let $\mathbb{N}$ denote the set of all natural numbers, and $\mathbb{Z}$ denote the set of all integers. Consider the functions $f : \mathbb{N} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{N}$ defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd,} \\ (4-n)/2 & \text{if } n \text{ is even,} \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$.

Then which of the following statements is (are) TRUE?

(A) $g \circ f$ is **NOT** one-one and $g \circ f$ is **NOT** onto

(B) $f \circ g$ is **NOT** one-one but $f \circ g$ is onto

(C) g is one-one and g is onto

(D) f is **NOT** one-one but f is onto

Answer (A, D)

Sol. $$f(n) = \begin{cases} \dfrac{n+1}{2}, & n = 2k+1 \\ \dfrac{4-n}{2}, & n = 2k \end{cases} = \begin{cases} k+1 & k \in N \cup \{0\} \\ 2-k, & k \in N \end{cases}$$

at $k = 0$, $f(1) = 1 = f(2) = 1 \Rightarrow f$ is not one-one

$2 - k$ covers all integers $\{1, 0, -1, ...\}$

$k + 1$ covers $\{1, 2, 3, ...\}$

$\Rightarrow$ $f(n)$ covers all integers $\Rightarrow f(n)$ is onto but not one-one

$$g(n) = \begin{cases} 3+2n, & n \geq 0 \\ -2n, & n < 0 \end{cases}$$

Notice that $1 \notin$ range of g(x) as $3 + 2n \neq 1, n \geq 0$

and $-2n \neq 1, n < 0$

$\Rightarrow$ g(n) is not onto

$$g(f(n)) = \begin{cases} 3+2f(n), & f(n) \geq 0 \\ -2f(n), & f(n) < 0 \end{cases} = \begin{cases} 3+2\left(\dfrac{n+1}{2}\right), & n = 2k+1 \\ 3+2\left(\dfrac{4-n}{2}\right), & n = 2k \end{cases}$$

$$= \begin{cases} n+4 & n = 2k+1 \\ 7-n, & n = 2k \end{cases}$$

$g(f(n))$ is always odd $\Rightarrow$ not onto

at $g(f(1)) = 5 = g(f(2)) \Rightarrow 0$ not one-one

$$f(g(n)) = \begin{cases} \dfrac{g(n)+1}{2}, & g(n) \text{ is odd natural} \\ \dfrac{4-g(n)}{2}, & g(n) \text{ is even natural} \end{cases} = \begin{cases} n+2, & n \ge 0 \\ n+2, & n < 0 \end{cases}$$

$\Rightarrow$ $(n+2)\ \forall\ n \Rightarrow f(g(n))$ is one-one

7. Let $\mathbb{R}$ denote the set of all real numbers. Let $z_1 = 1 + 2i$ and $z_2 = 3i$ be two complex numbers, where $i = \sqrt{-1}$. Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2|\}$.

Then which of the following statements is (are) TRUE?

(A) S is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$

(B) S is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$

(C) S is a circle with radius $\frac{\sqrt{2}}{3}$

(D) S is a circle with radius $\frac{2\sqrt{2}}{3}$

Answer (A, D)

Sol. $|x + iy - (1 + 2i)| = 2|x + iy - (3i)|$

$\Rightarrow$ $|x - 1 + (y - 2)i| = 2|x + (y - 3)i|$

$\Rightarrow$ $\sqrt{(x-1)^2 + (y-2)^2} = 2\sqrt{x^2 + (y-3)^2}$

$\Rightarrow$ $(x - 1)^2 + (y - 2)^2 = 4(x^2 + (y - 3)^2)$

$\Rightarrow$ $x^2 + y^2 - 2x - 4y + 5 = 4(x^2 + y^2 - 6y + 9)$

$\Rightarrow$ $3x^2 + 3y^2 + 2x - 20y + 31 = 0$

$\Rightarrow$ $x^2 + y^2 + \frac{2x}{3} - \frac{20y}{3} + \frac{31}{3} = 0$

$\Rightarrow$ Centre $\equiv \left(\frac{-1}{3}, \frac{10}{3}\right)$

Radius = $\sqrt{\left(\frac{-1}{3}\right)^2 + \left(\frac{10}{3}\right)^2 - \frac{31}{3}} = \sqrt{\frac{8}{9}}$

$= \frac{2\sqrt{2}}{3}$

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks	:	+4	If **ONLY** the correct numerical value is entered in the designated place;
Zero Marks	:	0	In all other cases.

8. Let the set of all relations R on the set $\{a, b, c, d, e, f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by $\mathcal{S}$.

Then the number of elements in $\mathcal{S}$ is ________________.

Answer (105.00)

Sol. Let $A = \{a, b, c, d, e, f\}$

$R \subset A \times A$, R is reflexive, $\Rightarrow (x, x) \in R \ \forall\ x \in R$

$\Rightarrow \left({}^6C_1\right) \Rightarrow$ 6 elements

We need 4 elements but since R is symmetric, we want to pairs (α, β) and (γ, δ) such that

$(\alpha, \beta) \in R$ and $(\gamma, \delta) \Rightarrow (\beta, \alpha) \in R$ and $(\delta, \gamma) \in R$

$\Rightarrow$ We need to choose the pairs

(α, β) and (γ, δ)

$\Rightarrow$ Total unordered pairs

$\Rightarrow {}^6C_2 = 15$ pairs

Out of these we need two pairs

$$\Rightarrow {}^{15}C_2 = \frac{15 \times 14}{2} = 105$$

9. For any two points M and N in the XY-plane, let $\overrightarrow{MN}$ denote the vector from M to N, and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY-plane. Let S be a point inside the triangle ΔPQR such that

$\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}$.

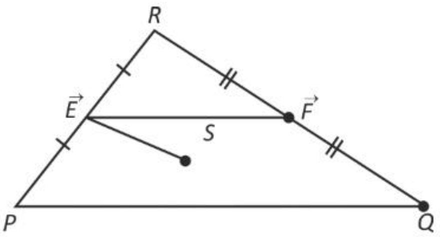
Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is ________.

Answer (01.20)

Sol.



$\overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SR}=\vec{0}$

$\vec{P}-\vec{S}+5(\vec{Q}-\vec{S})+6(\vec{R}-\vec{S})=0$

$\vec{P}+5\vec{Q}+6\vec{R}-12\vec{S}=0$

$|\overrightarrow{EF}|=\frac{1}{2}|\overrightarrow{PQ}|=|\vec{F}-\vec{E}|=\left|\frac{\vec{Q}+\vec{R}}{2}-\frac{\vec{R}+\vec{P}}{2}\right|=\left|\frac{\vec{P}-\vec{Q}}{2}\right|$

$|\overrightarrow{ES}|=|S-\vec{E}|=\left|\frac{\vec{P}+\vec{R}}{2}-\vec{S}\right|$

$=\left|\frac{\vec{P}+\vec{R}}{2}-\left(\frac{\vec{P}+5\vec{Q}+6\vec{R}}{12}\right)\right|$

$=\left|\frac{6\vec{P}+6\vec{R}-\vec{P}-5\vec{Q}-6\vec{R}}{12}\right|=\left|\frac{5\vec{P}-5\vec{Q}}{12}\right|$

$\Rightarrow \frac{|\overrightarrow{EF}|}{|\overrightarrow{ES}|}=\frac{\left|\frac{\vec{P}-\vec{Q}}{2}\right|}{\frac{5}{12}|\vec{P}-\vec{Q}|}=\frac{6}{5}=\frac{12}{10}=01.20$

10. Let S be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in S, but 0210222 is **NOT** in S.

Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x, is equal to__________.

Answer (762.00)

Sol. A : 0 exactly twice

B : 1 exactly twice

C : 0 and 1 exactly twice

$\Rightarrow n(A)={}^6C_2\cdot 2^5$ [0 can't be leftmost digit]

$= 480$

B : (1 at leftmost place) $\cup$ (1 is not at leftmost place)

$= {}^6C_1 \cdot 2^5 + {}^6C_2 . 2^4 = 432$

C : 0011222, no. of valid 7 digit number

$\Rightarrow \frac{7!}{3!2!2!} - \frac{6!}{3!2!} = 150$

$\Rightarrow$ Total numbers = 480 + 432 – 150

$\Rightarrow$ 762

11. Let α and β be the real numbers such that

$$\lim_{x\to 0} \frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt + \beta x \cos x\right) = 2.$$

Then the value of $\alpha + \beta$ is ___________.

Answer (02.40)

Sol. $\lim_{x\to 0} \frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt + \beta x \cos x\right) = 2 \quad \left(\left[\frac{0}{0}\right] \text{form}\right)$

$$\lim_{x\to 0} \frac{\frac{\alpha}{2}\cdot\left(\frac{1}{1-x^2}\right)\cdot 1 + \beta \cos x - \beta x \sin x}{3x^2}$$

$\Rightarrow \boxed{\frac{\alpha}{2} + \beta = 0}$

Again differentiate,

$$\lim_{x\to 0} \frac{\frac{\alpha}{2}(-1)(1-x^2)^{-2}(-2x) - \beta \sin x - \beta \sin x - \beta x \cos x}{6x}$$

Again differentiate,

$$\lim_{x\to 0} \frac{\alpha(-2)(1-x^2)^{-3}(-2x)^2 + \alpha(1-x^2)^{-2} - 2\beta \cos x - \beta \cos x + \beta x \sin x}{6}$$

$\Rightarrow \frac{\alpha - 2\beta - \beta}{6} = 2$

$\Rightarrow \alpha - 3\beta = 12$

$\alpha + 2\beta = 0$

$-5\beta = 12$

$\beta = \frac{-12}{5}$ & $\alpha = \frac{24}{5}$

$\alpha + \beta = \frac{12}{5} = 2.40$

12. Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$, and $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. Let the real numbers $a_1, a_2, \ldots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64f(a_{25})$ and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25} + 1),$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is_______________.

Answer (96.00)

Sol. $f(x) = a^x \forall x, y \in R$

$a_r = a_1 + (r - 1)d$. {Let d is common difference of A. P)

$$\sum_{i=1}^{50} f(a_i) = \sum_{i=1}^{50} a^{a_1 + (i-1)d}$$

$$= a^{a_1 - d} \sum_{i=1}^{50} a^{id} = a^{a_1 - d} = \frac{a^d(1 - (a^d)^{50})}{1 - a^d}$$

$$= a^{a_1 - d} . \frac{a^d(1 - a^{50d})}{1 - a^d}$$

$$\Rightarrow \quad a^{a_1} . \frac{(1 - a^{50d})}{1 - a^d} = 3(2^{25} + 1) \quad \ldots(i)$$

$f(a_{31}) = 64f(a_{25})$

$\Rightarrow \quad a^{a_1 + 30d} = 64. a^{a_1 + 24d}$

$\Rightarrow \quad a^{6d} = 64 = 2^6$

$a^d = 2 \quad \ldots(ii)$

Using (i) and (ii)

$$\Rightarrow \quad \frac{a^{a_1}(1 - 2^{50})}{1 - 2} = 3(2^{25} + 1)$$

$$\Rightarrow \quad a^{a_1}(2^{25}-1)(2^{25}+1)=3(2^{25}+1)$$

$$\Rightarrow \quad a^{a_1}=\frac{3}{2^{25}-1} \qquad \text{...(iii)}$$

Now, $\sum_{i=6}^{30} f(a_i)=a^{a_1-d}\sum_{i=6}^{30} a^{id}=a^{a_1-d}\cdot\frac{a^{6d}(a^{25d}-1)}{(a^d-1)}$

$$=a^{a_1}\cdot(a^d)^5\frac{|(a^d)^{25}-1|}{a^d-1}$$

$$=\frac{3}{2^{25}-1}\times\frac{2^5\cdot(2^{25}-1)}{2-1}$$

$= \boxed{96.00}$

13. For all $x > 0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx}-(\sin x)^2 y_1=0,\ y_1(1)=5,$$

$$\frac{dy_2}{dx}-(\cos x)^2 y_2=0,\ y_2(1)=\frac{1}{3},$$

$$\frac{dy_3}{dx}-\left(\frac{2-x^3}{x^3}\right)y_3=0,\ y_3(1)=\frac{3}{5e},$$

respectively. Then

$$\lim_{x\to 0^+}\frac{y_1(x)y_2(x)y_3(x)+2x}{e^{3x}\sin x}$$

is equal to ______________.

Answer (02.00)

Sol. $\frac{dy_1}{dx}-(\sin^2 x)y_1=0$

$$\Rightarrow \quad \int\frac{dy_1}{y_1}=\int\sin^2 x\,dx$$

$$\Rightarrow \quad \int\frac{dy_1}{y_1}=\int\frac{1-\cos 2x}{2}dx$$

$$\Rightarrow \quad \ln|y_1|=\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+C_1$$

$$\Rightarrow \quad y_1=e^{\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+C_1}$$

$\because$ $y_1(1) = 5$

$$\Rightarrow \quad C_1=\ln 5-\frac{1}{2}+\frac{\sin 2}{4}$$

$$\Rightarrow \quad y_1 = e^{\frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + \ln 5 - \frac{1}{2} + \frac{\sin 2}{4}} \qquad \ldots(1)$$

and $\dfrac{dy_2}{dx} = (\cos^2 x) y_2$

$$\Rightarrow \quad \int \frac{dy_2}{y_2} = \int \cos^2 x dx = \int \frac{1+\cos 2x}{2} dx$$

$$\Rightarrow \quad \ln|y_2| = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C_2$$

$\because$ $y_2(1) = \dfrac{1}{3}$

$$\Rightarrow \quad C_2 = -\ln 3 - \frac{1}{2} - \frac{\sin 2}{4}$$

$$\Rightarrow \quad y_2 = e^{\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) - \ln 3 - \frac{1}{2} - \frac{\sin 2}{4}} \qquad \ldots(2)$$

and $\dfrac{dy_3}{dx} = \left(\dfrac{2 - x^3}{x^3}\right) y_3$

$$\Rightarrow \quad \int \frac{dy_3}{y_3} = \int \left(\frac{2}{x^3} - 1\right) dx$$

$$\Rightarrow \quad \ln|y_3| = -\frac{1}{x^2} - x + C_3$$

$\because$ $y_3(1) = \dfrac{3}{5e}$

$$\Rightarrow \quad C_3 = 1 + \ln 3 - \ln 5$$

$$\Rightarrow \quad y_3 = e^{-\frac{1}{x^2} - x + 1 + \ln 3 - \ln 5} \qquad \ldots(3)$$

From eq^n (1), (2) and (3)

$$y_1(x)\ y_2(x)\ y_3(x) = e^{-\frac{1}{x^2}}$$

$$\therefore \quad \lim_{x \to 0^+} \frac{y_1(x) y_2(x) y_3(x) + 2x}{e^{3x} \sin x}$$

$$= \lim_{x \to 0^+} \frac{e^{\frac{-1}{x^2}} + 2x}{e^{3x} \sin x}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x} e^{\frac{-1}{x^2}} + 2}{e^{3x}\left(\frac{\sin x}{x}\right)} \qquad \because \lim_{x \to 0^+} \frac{1}{x} e^{\frac{-1}{x^2}} = 0$$

$$= \frac{0+2}{1 \times 1} = 2$$

SECTION 4 (Maximum Marks : 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

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14. Consider the following frequency distribution:

Value	4	5	8	9	6	12	11
Frequency	5	f_1	f_2	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.

For the given frequency distribution, let α denote the mean deviation about the mean, β denote the mean deviation about the median, and σ^2 denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		List-II
(P)	$7f_1 + 9f_2$ is equal to	(1)	146
(Q)	19α is equal to	(2)	47
(R)	19β is equal to	(3)	48
(S)	$19\alpha^2$ is equal to	(4)	145
		(5)	55

(A) (P) → (5) (Q) → (3) (R) → (2) (S) → (4)

(B) (P) → (5) (Q) → (2) (R) → (3) (S) → (1)

(C) (P) → (5) (Q) → (3) (R) → (2) (S) → (1)

(D) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)

Answer (C)

Sol.

Value (x_i)	f_i	cf	f_ix_i
4	5	5	20
5	f_1	$5 + f_1$	20
6	1	$6 + f_1$	6
8	f_2	$6 + f_1 + f_2$	24
9	2	$8 + f_1 + f_2$	18
11	3	$11 + f_1 + f_2$	33
12	1	$12 + f_1 + f_2$	12
	$12 + f_1 + f_2$		$\Sigma f_ix_i = 133$

Given $12 + f_1 + f_2 = 19$

$\Rightarrow f_1 + f_2 = 7$

Median = 6

Median = $\left(\frac{N+1}{2}\right)^{th}$ term $= \left(\frac{19+1}{2}\right)^{th}$ term = 10^{th} term

$\Rightarrow 6 + f_1 = 10 \Rightarrow f_1 = 4$ and $f_2 = 3$

$\therefore 7f_1 + 9f_2 = 7 \times 4 + 9 \times 3$

$= 55 \Rightarrow P \to 5$

Mean $(\bar{X}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{133}{19} = 7$

Mean deviation about mean $(\alpha) = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i}$

$= \frac{5(3) + 4(2) + 1(1) + 3(1) + 2(2) + 3(4) + 1(5)}{19} = \frac{48}{19}$

$\Rightarrow 19\alpha = 48 \quad Q \to 3$

Mean deviation about median (β)

$= \frac{\Sigma f_i |x_i - M|}{\Sigma f_i}$

$= \frac{5(2) + 4(1) + 1(0) + 3(2) + 2(3) + 3(5) + 1(6)}{19}$

$= \frac{47}{19}$

$19\beta = 47$ $\quad$ R $\to$ 2

Variance $(\sigma^2) = \frac{\Sigma f_i f_i^2}{\Sigma f_i} - (\bar{X})^2$

$= \frac{5\times16+4\times25+1\times36+3\times64+2\times81+3\times121+1\times144}{19} - (7)^2$

$= \frac{1077}{19} - 49 = \frac{146}{19}$

$\Rightarrow\ 19\sigma^2 = 146$

$S \to 1$

$P \to 5,\ Q \to 3,\ R \to 2,\ S \to 1$

15. Let $\mathbb{R}$ denote the set of all real numbers. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let n denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		**List-II**
(P)	The minimum value of n for which the function $f(x) = \left[\frac{10x^3 - 45x^2 + 60x + 35}{n}\right]$ is continuous on the interval [1, 2], is	(1)	8
(Q)	The minimum value of n for which $g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$, $x \in \mathbb{R}$, is an increasing function on $\mathbb{R}$, is	(2)	9
(R)	The smallest natural number n which is greater than 5, such that $x = 3$ is a point of local minima of $h(x) = (x^2 - 9)^n (x^2 + 2x + 3)$, is	(3)	5
(S)	Number of $x_0 \in \mathbb{R}$ such that $l(x) = \sum_{k=0}^{4}\left(\sin\lvert x-k\rvert + \cos\left\lvert x - k + \frac{1}{2}\right\rvert\right)$, $x \in \mathbb{R}$, is **NOT** differentiable at x_0, is	(4)	6
		(5)	10

(A) (P) → (1) (Q) → (3) (R) → (2) (S) → (5)

(B) (P) → (2) (Q) → (1) (R) → (4) (S) → (3)

(C) (P) → (5); (Q) → (1); (R) → (4); (S) → (3)

(D) (P) → (2); (Q) → (3); (R) → (1); (S) → (5)

Answer (B)

Sol. (P) Let $g(x) = 10x^3 - 45x^2 + 60x + 35$

$g'(x) = 30(x^2 - 3x + 2) = 30(x - 1)(x - 2)$

$\Rightarrow g(x)$ is decreasing in [1, 2]

Now, $f(1) = \left[\frac{g(1)}{n}\right] = \left[\frac{60}{n}\right]$, $f(2) = \left[\frac{55}{n}\right]$

For $f(x)$ to be continuous in [1, 2] its integral value should remain same in whole interval for $n = 9$, $f(1) = 6$, $f(2) = 6$

(P) $\rightarrow$ (2)

(Q) $g'(x) = (2n^2 - 13n - 15)(3x^2 + 3)$

$= (2n - 15)(n + 1)(3x^2 + 3) > 0$

for $g(x)$ to be increasing min $n = 8$

$\Rightarrow Q \rightarrow 1$ matching

(R) $h(x) = (x^2 - 9)^n(x^2 + 2x + 3)$

$h(3) = 0$ for $n > 5$

at $n = 6 \Rightarrow n(3 + \delta) > h(3)$

$n(3 - \delta) > n(3)$

$\Rightarrow h(x)$ has local minima at $x = 3$ for $n = 6$

R $\rightarrow$ 4 matching

(S) $l(x) = \sin|x| + \cos\left|x + \frac{1}{2}\right| + \sin|x - 1| + \cos\left|x - \frac{1}{2}\right|$

$+ \ldots\ldots + \sin|x - 4| + \cos\left|x - \frac{7}{2}\right|$

as $\sin|x - a|$ is non differentiable at $x = a$

but $\cos|x - a|$ remains differentiable at $x = a$

$\Rightarrow$ given $l(x)$ is non-differentiable at

$x_0 = 0, 1, 2, 3, 4$ (5 points)

(S) → (3) matching

(P) → (2) (Q) → (1) (R) → (4) (S) → (3)

16. Let $\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$, and $\vec{u}$ and $\vec{v}$ be two vectors, such that $\vec{u} \times \vec{v} = \vec{w}$ and $\vec{v} \times \vec{w} = \vec{u}$. Let α, β, γ, and t be real numbers such that

$\vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $-t\alpha + \beta + \gamma = 0$, $\alpha - t\beta + \gamma = 0$, and $\alpha + \beta - t\gamma = 0$.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		**List-II**
(P)	$\lvert\vec{v}\rvert^2$ is equal to	(1)	0
(Q)	If $\alpha = \sqrt{3}$, then γ^2 is equal to	(2)	1
(R)	If $\alpha = \sqrt{3}$, then $(\beta + \gamma)^2$ is equal to	(3)	2
(S)	If $\alpha = \sqrt{2}$, then $t + 3$ is equal to	(4)	3
		(5)	5

(A) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)

(B) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (3)

(D) (P) → (5) (Q) → (4) (R) → (1) (S) → (3)

Answer (A)

Sol. $\vec{u} \times \vec{v} = \vec{w}$ and $\vec{v} \times \vec{w} = \vec{u}$

⇒ $\vec{u}$, $\vec{v}$ and $\vec{w}$ are mutually perpendicular.

$(\vec{v} \times \vec{w}) \times \vec{v} = \vec{w}$

⇒ $\vec{w}(\vec{v} \cdot \vec{v}) - \vec{v}(\vec{v} \cdot \vec{w}) = \vec{w}$

⇒ $\vec{w}(\lvert\vec{v}\rvert^2 - 1) - \vec{v}(\vec{v} \cdot \vec{w}) = \vec{0}$

$\Rightarrow \quad |\vec{v}|^2 = 1$ and $\vec{v}\cdot\vec{w} = 0$

$|\vec{u}||\vec{v}| = |\omega| \Rightarrow |\vec{u}| = \sqrt{6}$

$\vec{u}\cdot\vec{w} = 0 \Rightarrow \alpha + \beta - 2\gamma = 0$

and $-t\alpha + \beta + \gamma = 0$ ...(i)

$\alpha - t\beta + \gamma = 0$ ....(ii)

$\alpha + \beta - t\gamma = 0$ ...(iii)

(ii) – (i) $\Rightarrow \quad \alpha(1 + t) = (t + 1)\beta$

(iii) – (ii) $\Rightarrow \quad \beta(1 + t) = (1 + t)\gamma$

$\Rightarrow \quad \alpha(1 + t) = \beta(1 + t) = \gamma(1 + t)$

either $t = -1$ or $\alpha = \beta = \gamma$

$\Rightarrow \quad \sqrt{\alpha^2 + \alpha^2 + \alpha^2} = \sqrt{6}$

$\Rightarrow \quad \alpha = \sqrt{2}, -t\alpha = -\alpha - \alpha$

$\Rightarrow \quad t = 2$

Since $\alpha = \sqrt{3}$

$\Rightarrow \quad t = -1$

$\Rightarrow \quad \alpha + \beta + \gamma = 0$

$\alpha + \beta - 2\gamma = 0$

$\Rightarrow \quad \gamma = 0$

(Q) $\rightarrow$ 1

$\alpha + \beta = 0 \Rightarrow (\beta + \gamma) = -\alpha \Rightarrow (\beta + \gamma)^2 = \alpha^2 = 3$, (R) $\rightarrow$ 4

$\Rightarrow \quad |\vec{v}|^2 = 1 \rightarrow$ (P) $\rightarrow$ 2

If $\alpha = \sqrt{3} \Rightarrow \gamma^2 = 0 \rightarrow$ (Q) $\rightarrow$ 1

If $\alpha = \sqrt{3} \Rightarrow (\beta + \gamma)^2 = (\sqrt{3})^2 = 3$, (R) $\rightarrow$ 4

If $\alpha = \sqrt{2}, t + 3 = (2) + 3 = 5$, (S) $\rightarrow$ 5

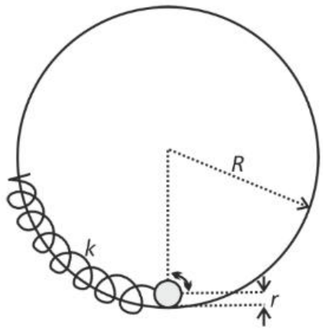
PART-II : PHYSICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. The center of a disk of radius r and mass m is attached to a spring of spring constant k, inside a ring of radius $R > r$ as shown in the figure. The other end of the spring is attached on the periphery of the ring. Both the ring and the disk are in the same vertical plane. The disk can only roll along the inside periphery of the ring, without slipping. The spring can only be stretched or compressed along the periphery of the ring, following the Hooke's law. In equilibrium, the disk is at the bottom of the ring. Assuming small displacement of the disc, the time period of oscillation of center of mass of the disk is written as $T = \frac{2\pi}{\omega}$. The correct expression for ω is (g is the acceleration due to gravity):



(A) $\sqrt{\frac{2}{3}\left(\frac{g}{R-r}+\frac{k}{m}\right)}$

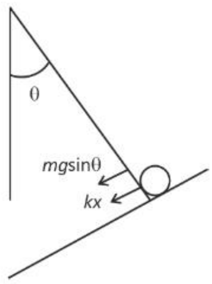
(B) $\sqrt{\frac{2g}{3(R-r)}+\frac{k}{m}}$

(C) $\sqrt{\frac{1}{6}\left(\frac{g}{R-r}+\frac{k}{m}\right)}$

(D) $\sqrt{\frac{1}{4}\left(\frac{g}{R-r}+\frac{k}{m}\right)}$

Answer (A)

Sol.



$$\tau = rmg\sin\theta + k(R-r)\theta r = \frac{3}{2}mr^2\alpha'$$

$$r\theta' = (R-r)\theta$$

$$\theta' = \left(\frac{R-r}{r}\right)\theta$$

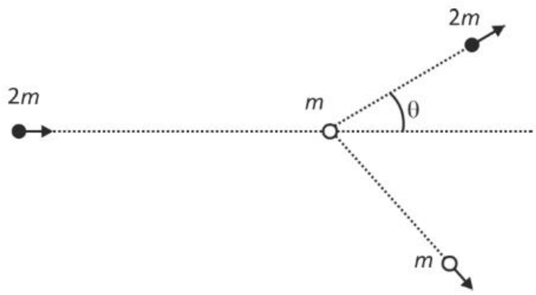
$$\alpha' = \left(\frac{R-r}{r}\right)\alpha$$

$$(rmg + k(R-r)r]\theta = \frac{3}{2}mr(R-r)\alpha$$

$$\alpha = \omega^2\theta$$

$$\omega = \sqrt{\frac{2}{3}\left[\frac{k}{m} + \frac{g}{(R-r)}\right]}$$

2. In a scattering experiment, a particle of mass $2m$ collides with another particle of mass m, which is initially at rest. Assuming the collision to be perfectly elastic, the maximum angular deviation θ of the heavier particle, as shown in the figure, in radians is:



(A) π

(B) $\tan^{-1}\left(\frac{1}{2}\right)$

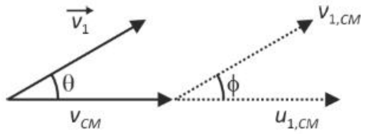
(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Answer (D)

Sol. **Before collision** 

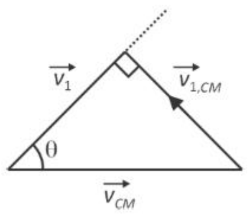
After collision



In centre of mass frame

Speed does not change only orientation does

For θ to maximize



$\overrightarrow{v_{1,CM}} \perp \vec{v_1}$

$\Rightarrow \sin\theta = \dfrac{v_{1,CM}}{v_{CM}}$

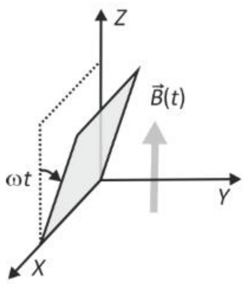
$v_{1,CM} = \dfrac{m_2 \overrightarrow{v_{12}}}{m_1 + m_2} \Rightarrow \vec{v_1} = \dfrac{m_2 \vec{u_1}}{m_1 + m_2}$

$v_{CM} = \dfrac{m_1 \vec{v_1} + m_2 \vec{v_2}}{m_1 + m_2} = \dfrac{m_1 \vec{u_1}}{m_1 + m_2}$

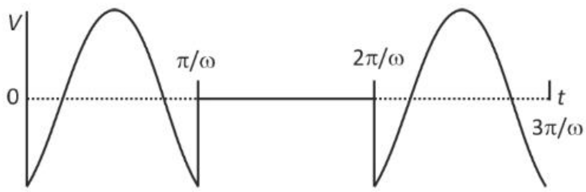
$\Rightarrow \sin\theta = \dfrac{m_2}{m_1} \Rightarrow \sin\theta = \dfrac{m}{2m}$

$\theta = 30^\circ = \dfrac{\pi}{6}$

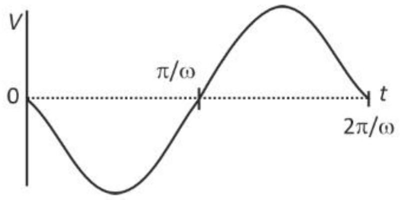
3. A conducting square loop initially lies in the XZ plane with its lower edge hinged along the X-axis. Only in the region $y \geq 0$, there is a time dependent magnetic field pointing along the Z-direction, $\vec{B}(t) = B_0(\cos\omega t)\hat{k}$, where B_0 is a constant. The magnetic field is zero everywhere else. At time $t = 0$, the loop starts rotating with constant angular speed ω about the X axis in the clockwise direction as viewed from the $+X$ axis (as shown in the figure). Ignoring self-inductance of the loop and gravity, which of the following plots correctly represents the induced e.m.f. (V) in the loop as a function of time:



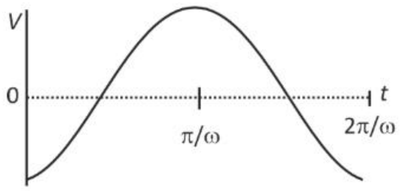
(A)



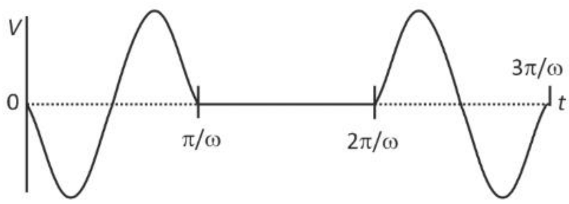
(B)



(C)



(D)



Answer (A)

Sol. $\phi = B_0 \cos(\omega t) l^2 \sin(\omega t)$

$$\phi = \frac{B_0 l^2}{2} \sin(2\omega t)$$

$$\sum = \left|\frac{d\phi}{dt}\right| = B_0 \omega l^2 \cos(2\omega t) \text{ only for half rotation}$$

4. Figure 1 shows the configuration of main scale and Vernier scale before measurement. Fig. 2 shows the configuration corresponding to the measurement of diameter D of a tube. The measured value of D is:

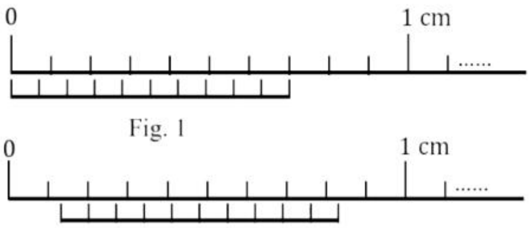


Fig. 1
Fig. 2

(A) 0.12 cm

(B) 0.11 cm

(C) 0.13 cm

(D) 0.14 cm

Answer (C)

Sol. For (VS) 10 div = 7 mm

1 div = 0.7 mm

Reading: main scale = 1 mm

VS 1 marking matches with main scale div

So VS reads (1 – 0.7 mm) = 0.3 mm

So total = 1 + 0.3 mm = 1.3 mm

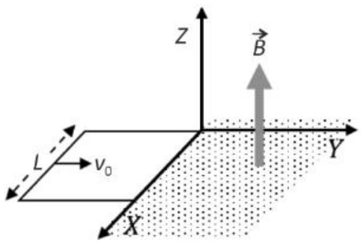
$\Rightarrow$ 0.13 cm

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

5. A conducting square loop of side L, mass M and resistance R is moving in the XY plane with its edges parallel to the X and Y axes. The region $y \geq 0$ has a uniform magnetic field, $\vec{B} = B_0\hat{k}$. The magnetic field is zero everywhere else. At time $t = 0$, the loop starts to enter the magnetic field with an initial velocity $v_0\hat{j}$ m/s, as shown in the figure. Considering the quantity $K = \frac{B_0^2 L^2}{RM}$ in appropriate units, ignoring self-inductance of the loop and gravity, which of the following statements is/are correct:



(A) If $v_0 = 1.5KL$, the loop will stop before it enters completely inside the region of magnetic field

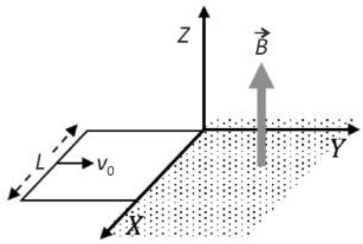
(B) When the complete loop is inside the region of magnetic field, the net force acting on the loop is zero.

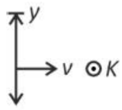
(C) If $v_0 = \frac{KL}{10}$, the loop comes to rest at $t = \left(\frac{1}{K}\right)\ln\left(\frac{5}{2}\right)$

(D) If $v_0 = 3KL$, the complete loop enters inside the region of magnetic field at time $t = \left(\frac{1}{K}\right)\ln\left(\frac{3}{2}\right)$

Answer (B, D)

Sol.





$$K = \frac{B_0^2 L^2}{RM}$$

$$\varepsilon = vBL$$

$$I = \frac{vBL}{R}$$

$$F_{(\text{drag})} = -\frac{vBL}{R} \cdot LB$$

$$m\frac{dv}{dt} = -\frac{(B^2L^2)v}{R}$$

$$m\frac{dv}{dx} \cdot v = -\frac{B^2L^2}{R} \cdot v \qquad \Rightarrow \quad mdv = -\frac{B^2L^2}{R} \cdot dx$$

$$m|v|_{v_0}^{v} = \frac{-B^2L^2}{R} \cdot x|_0^x$$

$$\Rightarrow \quad m(v - v_0) = \frac{-B^2L^2x}{R}$$

$$\Rightarrow \quad v = v_0 - \frac{B^2L^2}{mR} \cdot x$$

If $v_0 = 1.5KL$ then for $x = L$

$$v = \frac{3}{2}KL - KL = \frac{1}{2}KL$$

So, the loop will have some velocity and it will not stop.

If the loop completely gets inside then further no current flows and no force acts.

If $v_0 = \frac{KL}{10}$ then for $v = 0$

$$\frac{dv}{dt} = -\frac{B^2L^2}{mR}v$$

$$\int_{v_0}^{v} \frac{dv}{v} = \int_0^t -K \cdot dt \qquad \Rightarrow \qquad \ln\frac{v}{v_0} = -Kt$$

$\frac{v}{v_0} = e^{-Kt}$ it will never stop

If $v_0 = 3KL$ then

v at $x = L$ is $v = 3KL - KL = 2KL$

$$\Rightarrow \quad \frac{2}{3} = e^{-Kt} \qquad \Rightarrow \qquad \frac{1}{K}\ln\left(\frac{3}{2}\right) = t$$

6. Length, breadth and thickness of a strip having a uniform cross section are measured to be 10.5 cm, 0.05 mm, and 6.0 μm, respectively. Which of the following option(s) give(s) the volume of the strip in cm^3 with correct significant figures :

(A) 3.2×10^{-5} (B) 32.0×10^{-6}

(C) 3.0×10^{-5} (D) 3×10^{-5}

Answer (D)

Sol. $l = 10.5$ cm $\rightarrow$ 3 SF

$b = 0.05$ mm $= 5 \times 10^{-3}$ cm $\rightarrow$ 1 SF

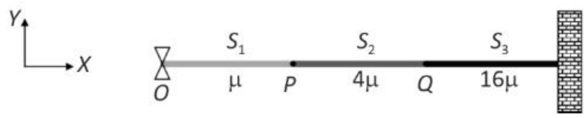
$h = 6.0$ μm $\Rightarrow 6 \times 10^{-4}$ cm $\rightarrow$ 2 SF

$v = 315 \times 10^{-7}$

$v = 3.15 \times 10^{-5}$

$v = 3 \times 10^{-5}$ 1 SF

7. Consider a system of three connected strings, S_1, S_2 and S_3 with uniform linear mass densities μ kg/m, 4μ kg/m and 16μ kg/m, respectively, as shown in the figure. S_1 and S_2 are connected at the point P, whereas S_2 and S_3 are connected at the point Q, and the other end of S_3 is connected to a wall. A wave generator O is connected to the free end of S_1. The wave from the generator is represented by $y = y_0 \cos(\omega t - kx)$ cm, where y_0, ω and k are constants of appropriate dimensions. Which of the following statements is/are correct :



(A) When the wave reflects from P for the first time, the reflected wave is represented by $y = \alpha_1 y_0 \cos(\omega t + kx + \pi)$ cm, where α_1 is a positive constant

(B) When the wave transmits through P for the first time, the transmitted wave is represented by $y = \alpha_2 y_0 \cos(\omega t - kx)$ cm, where α_2 is a positive constant

(C) When the wave reflects from Q for the first time, the reflected wave is represented by $y = \alpha_3 y_0 \cos(\omega t - kx + \pi)$ cm, where α_3 is a positive constant

(D) When the wave transmits through Q for the first time, the transmitted wave is represented by $y = \alpha_4 y_0 \cos(\omega t - 4kx)$ cm, where α_4 is a positive constant

Answer (A, D)

Sol. Reflection of wave in string from fixed end, reflected wave phase change by π.

And reflected wave from free (rarer) end No phase change.

$\because \quad \omega = kV$

$\omega = k\sqrt{\frac{T}{\mu}}$ given $y = y_0 \cos(\omega t - kx)$ cm

$\frac{\sqrt{T}}{\mu} = \text{const.}$ For medium $PQ(S_2)$

$k_{S_2} = k_{PQ} = 2k$

For medium S_3

$k_{S_3} = 4k$

Option (A) reflection from P, phase change by π,

No change in 'k'. Hence correct option.

In option (B) $k_{S_2} = 2k$ but in option given $k_S = k$

Hence wrong.

In option (C) direction of reflected wave in negative direction.

Hence wrong.

Option (D) no phase change in transmitted wave and given $k_{S_3} = 4k$

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	If ONLY the correct numerical value is entered in the designated place;
Zero Marks	:	0	In all other cases.

8. A person sitting inside an elevator performs a weighing experiment with an object of mass 50 kg. Suppose that the variation of the height y (in m) of the elevator, from the ground, with time t (in s) is given by $y = 8\left[1+\sin\left(\frac{2\pi t}{T}\right)\right]$, where $T = 40\pi$ s. Taking acceleration due to gravity, $g = 10\ \text{m/s}^2$, the maximum variation of the object's weight (in N) as observed in the experiment is ________

Answer (02.00)

Sol. $y = 8(1 + \sin(\omega t))$,

Acceleration of elevator

$$\omega = \frac{2\pi}{40\pi} = \frac{1}{20}$$

$$a = -8\omega^2 \sin(\omega t)$$

$$|a_{max}| = 8 \times \frac{1}{400} = \frac{1}{50}$$

$$W_{max} = m(g + a_{max})$$

$$W_{min} = m(g - a_{max})$$

$$\Delta W = 2ma_{max}$$

$$= 2 \times 50 \times \frac{1}{50}$$

$= 2$ N

9. A cube of unit volume contains 35×10^7 photons of frequency 10^{15} Hz. If the energy of all the photons is viewed as the average energy being contained in the electromagnetic waves within the same volume, then the amplitude of the magnetic field is $\alpha \times 10^{-9}$ T. Taking permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ Tm/A, Planck's constant $h = 6 \times 10^{-34}$ Js and $\pi = \frac{22}{7}$, the value of α is ________

Answer (22.98)

Sol. $E = N \cdot hf$

$E = 35 \times 10^7 \times 6 \times 10^{-34} \times 10^{15}$

$$\frac{B_0^2}{2\mu_0} = E$$

$$B_0^2 = 2 \times 4 \times \frac{22}{7} \times 10^{-7} \times 35 \times 10^7 \times 6 \times 10^{-34} \times 10^{15}$$

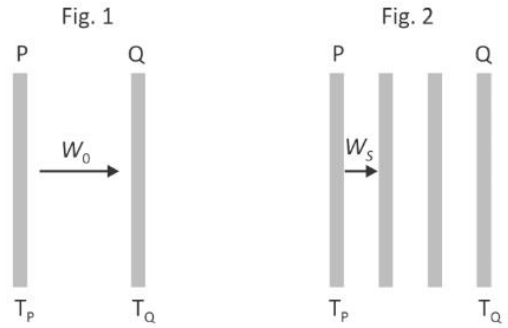
$= 5280 \times 10^{-19}$

$B_0^2 = 528 \times 10^{-18}$

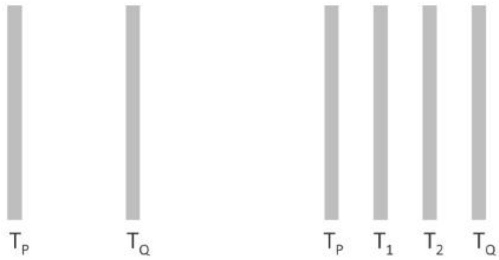
$B_0 = 22.978 \times 10^{-9}$

$\alpha = 22.98$

10. Two identical plates P and Q, radiating as perfect black bodies, are kept in vacuum at constant absolute temperatures T_P and T_Q, respectively, with $T_Q < T_P$, as shown in Fig. 1. The radiated power transferred per unit area from P to Q is W_0. Subsequently, two more plates, identical to P and Q, are introduced between P and Q, as shown in Fig. 2. Assume that heat transfer takes place only between adjacent plates. If the power transferred per unit area in the direction from P to Q (Fig. 2) in the steady state is W_S, then the ratio $\frac{W_0}{W_S}$ is ________



Answer (03.00)



$W_0 = K\left(T_P^4 - T_Q^4\right)$ $\qquad$ $T_1 > T_2$

In the steady state

$$W_S = K\left(T_P^4 - T_1^4\right)$$

T_1 and T_2 will remain constant

$$\Rightarrow \left(T_P^4 - T_1^4\right) = \left(T_1^4 - T_2^4\right) = \left(T_2^4 - T_P^4\right)$$

$$\Rightarrow \quad T_P^4 + T_2^4 = 2T_1^4 \qquad ...(i)$$

$$T_1^4 + T_Q^4 = 2T_2^4 \qquad ...(ii)$$

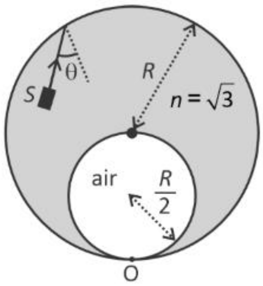
2(i) + (ii)

$$\Rightarrow \quad T_1^4 = \frac{2T_P^4 + T_Q^4}{3}$$

$$W_S = K\left[T_P^4 - \frac{\left(2T_P^4 + T_Q^4\right)}{3}\right] = \frac{K\left(T_P^4 - T_Q^4\right)}{3}$$

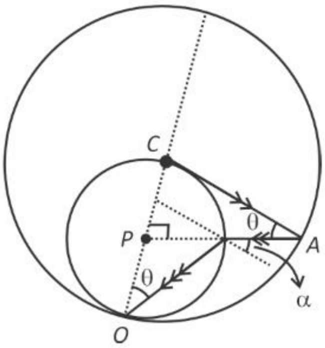
$$\boxed{\frac{W_0}{W_S} = 3}$$

11. A solid glass sphere of refractive index $n = \sqrt{3}$ and radius R contains a spherical air cavity of radius $\frac{R}{2}$, as shown in the figure. A very thin glass layer is present at the point O so that the air cavity (refractive index $n = 1$) remains inside the glass sphere. An unpolarized, unidirectional and monochromatic light source S emits a light ray from a point inside the glass sphere towards the periphery of the glass sphere. If the light is reflected from the point O and is fully polarized, then the angle of incidence at the inner surface of the glass sphere is θ. The value of $\sin\theta$ is ________



Answer (00.75)

Sol.



θ = 60° because it is brewster angle

At point *X*

Snell's law gives $\alpha = 30°$

$\Rightarrow$ *AP* is perpendicular to *CO*

Therefore in ΔCPA

$$\sin\theta = \frac{CP}{CA} = \frac{3R/4}{R} = \frac{3}{4}$$

12. A single slit diffraction experiment is performed to determine the slit width using the equation, $\frac{bd}{D} = m\lambda$, where *b* is the slit width, *D* the shortest distance between the slit and the screen, *d* the distance between the m^{th} diffraction maximum and the central maximum, and λ is the wavelength. *D* and *d* are measured with scales of least count of 1 cm and 1 mm, respectively. The values of λ and *m* are known precisely to be 600 nm and 3, respectively. The absolute error (in μm) in the value of *b* estimated using the diffraction maximum that occurs for *m* = 3 with *d* = 5 mm and *D* = 1 m is

Answer (94.50)

Sol. $b = \dfrac{m\lambda D}{d}$

$$b = \frac{3 \times 600 \times 1}{5} \times 10^{-6}$$

Case 1 with single dash and case 2 with double dash

$$\begin{cases} b' = \dfrac{3 \times 600 \times 1.01}{4} \times 10^{-6} & \dfrac{N^+}{D^-} \\ b'' = \dfrac{3 \times 600 \times 0.99}{6} \times 10^{-6} & \dfrac{N^-}{D^+} \end{cases}$$

$$\begin{cases} \dfrac{b'}{b} = \dfrac{5.05}{4} \\ \dfrac{b''}{b} = \dfrac{4.95}{6} \end{cases}$$

$$\begin{cases} (\Delta b)_1 = \left(\dfrac{5.05}{4} - 1\right) b \\ \qquad = \dfrac{1.05}{4} b \\ (\Delta b)_2 = \left(\dfrac{4.95}{6} - 1\right) b = \dfrac{1.05}{6} b \end{cases}$$

$$\Delta b_1 = \frac{1.05}{4} \times \frac{3 \times 600}{5} \times 10^{-6}$$

$$= \frac{90 \times 1.05 \times 10^{-6}}{9450}$$

$$\Delta b_1 = 94.50\ \mu\text{m}$$

13. Consider an electron in the $n = 3$ orbit of a hydrogen-like atom with atomic number Z. At absolute temperature T, a neutron having thermal energy $k_B T$ has the same de Broglie wavelength as that of this electron. If this temperature is given by $T = \dfrac{z^2 h^2}{\alpha \pi^2 a_0^2 m_N k_B}$, (where h is the Planck's constant, k_B is the Boltzmann constant, m_N is the mass of the neutron and a_0 is the first Bohr radius of hydrogen atom) then the value of α is _______

Answer (72.00)

Sol. $2\pi r_3 = 3\lambda_e$

$$\lambda_e = \frac{2\pi r_3}{3}$$

$$\lambda_n = \frac{h}{p} = \frac{h}{\sqrt{2km_n}}$$

$$\frac{2\pi r_3}{3} = \frac{h}{\sqrt{2K_B T m_n}}$$

$$r_3 = \frac{a_0 \times (3)^2}{Z}$$

$$\frac{6\pi a_0}{Z} = \frac{h}{\sqrt{2K_B T m_n}}$$

$$T = \frac{h^2 Z^2}{36\pi^2 a_0^2 \times 2K_B m_n}$$

$$= \frac{h^2 Z^2}{72\pi^2 a_0^2 K_B m_n}$$

$\alpha = 72$

SECTION 4 (Maximum Marks : 12)

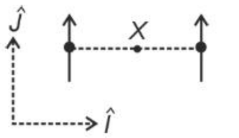
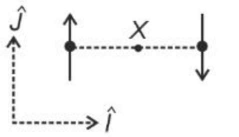
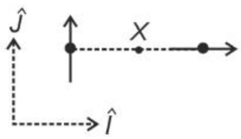
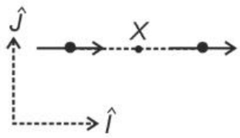
- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

---

14. List-I shows four configurations, each consisting of a pair of ideal electric dipoles. Each dipole has a dipole moment of magnitude p, oriented as marked by arrows in the figures. In all the configurations the dipoles are fixed such that they are at a distance $2r$ apart along the x direction. The midpoint of the line joining the two dipoles is X. The possible resultant electric fields $\vec{E}$ at X are given in List-II.

Choose the option that describes the correct match between the entries in **List-I** to those in **List-II.**

	List-I		**List-II**
(P)		(1)	$\vec{E}=0$
(Q)		(2)	$\vec{E}=-\frac{p}{2\pi\varepsilon_0 r^3}\hat{j}$
(R)		(3)	$\vec{E}=-\frac{p}{4\pi\varepsilon_0 r^3}(\hat{i}-\hat{j})$
(S)		(4)	$\vec{E}=\frac{p}{4\pi\varepsilon_0 r^3}(2\hat{i}-\hat{j})$
		(5)	$\vec{E}=\frac{p}{\pi\varepsilon_0 r^3}\hat{i}$

(A) P→3, Q→1, R→2, S→4

(B) P→4, Q→5, R→3, S→1

(C) P→2, Q→1, R→4, S→5

(D) P→2, Q→1, R→3, S→5

Answer (C)

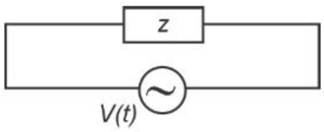
Sol. (P) $E = \frac{-2kp}{r^3}\hat{j} = -\frac{p}{2\pi\varepsilon_0 r^3}\hat{j}$, P→(2)

(Q) $E = 0$, Q→(1)

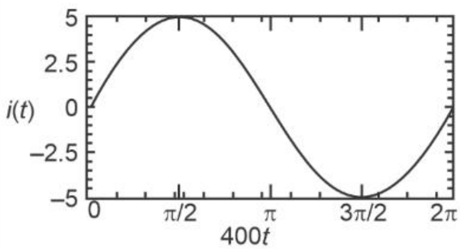
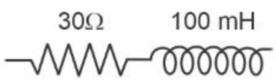
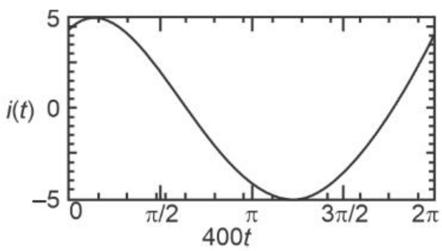
(R) $E = \frac{2kp}{r^3}\hat{i} - \frac{kp}{r^3}\hat{j}$, R→(4)

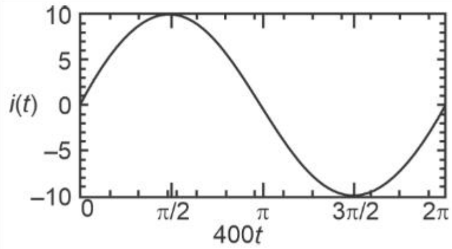
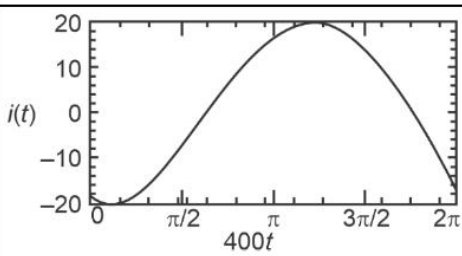
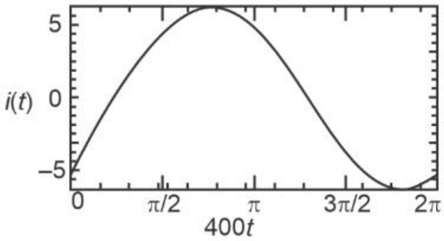
(S) $E = \frac{4kp}{r^3}\hat{i}$, S→(5)

15. A circuit with an electrical load having impedance Z is connected with an AC source as shown in the diagram. The source voltage varies in time as $V(t) = 300\sin(400t)$ V, where t is time in s. List-I shows various options for the load. The possible currents $i(t)$ in the circuit as a function of time are given in **List-II**.



Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

	List-I		**List-II**
(P)		(1)	
(Q)		(2)	

(R)	50 μF 30Ω 25 mH 	(3)	
(S)	50 μF 60Ω 125 mH 	(4)	
		(5)	

(A) P→3, Q→5, R→2, S→1

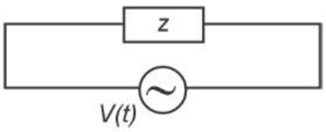
(B) P→1, Q→5, R→2, S→3

(C) P→3, Q→4, R→2, S→1

(D) P→1, Q→4, R→2, S→5

Answer (A)

Sol. $V(t) = 300 \sin(400t)$

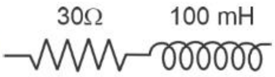


(P) 30Ω

$\Rightarrow \quad i_0 = \frac{300}{30} = 10 \text{ A}$

and $\phi = 0$

$\Rightarrow$ (P)→(3)

(Q) 

$$\Rightarrow \quad Z = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{(30)^2 + \left(400 \times 100 \times 10^{-3}\right)^2} = 50\ \Omega$$

$$\therefore \quad i_0 = \frac{300}{50} = 6\ \text{A}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = 53^\circ\ \text{(lag)}$$

(Q)→(5)

(R) $$Z = \sqrt{(30)^2 + \left(\frac{1}{400 \times 50 \times 10^{-6}} - 400 \times 25 \times 10^{-3}\right)^2}$$

$$= \sqrt{(30)^2 + (40)^2} = 50\ \Omega$$

∴ R→(2)

(S) $$Z = \sqrt{(60)^2 + \left(\frac{1}{50 \times 10^{-6} \times 400} - 125 \times 10^{-3} \times 400\right)}$$

$$= \sqrt{(60)^2 + (50 - 50)^2}$$

$= 60\ \Omega$

$$\therefore \quad i_0 = \frac{300}{60} = 5\ \text{A}$$

(S)→1

16. List-I shows various functional dependencies of energy (E) on the atomic number (Z). Energies associated with certain phenomena are given in List-II.

Choose the option that describes the correct match between the entries in **List-I** to those in **List-II.**

	List-I		List-II
(P)	$E \propto Z^2$	(1)	energy of characteristic x-rays
(Q)	$E \propto (Z-1)^2$	(2)	electrostatic part of the nuclear binding energy for stable nuclei with mass numbers in the range 30 to 170
(R)	$E \propto Z(Z-1)$	(3)	energy of continuous x-rays
(S)	E is practically independent of Z	(4)	average nuclear binding energy per nucleon for stable nuclei with mass number in the range 30 to 170
		(5)	energy of radiation due to electronic transitions from hydrogen-like atoms

(A) P→4, Q→3, R→1, S→2

(B) P→5, Q→2, R→1, S→4

(C) P→5, Q→1, R→2, S→4

(D) P→3, Q→2, R→1, S→5

Answer (C)

Sol. ∵ $E \propto Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

⇒ (P)→(5)

∴ $E \propto (Z-1)^2$

⇒ Energy of characteristic x-rays.

⇒ (Q)→(1)

Also, $E \propto Z(Z-1)$

Electrostatic energy of proton in nucleus.

⇒ (R)→(2)

(S) E is independent of Z

⇒ Average binding energy per nucleon in range of 30 to 170.

⇒ (S)→(4)

PART-III : CHEMISTRY

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. The heating of NH_4NO_2 at 60–70ºC and NH_4NO_3 at 200–250ºC is associated with the formation of nitrogen containing compounds **X** and **Y**, respectively. **X** and **Y**, respectively, are

(A) N_2 and N_2O (B) NH_3 and NO_2

(C) NO and N_2O (D) N_2 and NH_3

Answer (A)

Sol. $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

$NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$

2. The correct order of the wavelength maxima of the absorption band in the ultraviolet-visible region for the given complexes is

(A) $[Co(CN)_6]^{3-} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_5(H_2O)]^{3+} < [Co(NH_3)_5(Cl)]^{2+}$

(B) $[Co(NH_3)_5(Cl)]^{2+} < [Co(NH_3)_5(H_2O)]^{3+} < [Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-}$

(C) $[Co(CN)_6]^{3-} < [Co(NH_3)_5(Cl)]^{2+} < [Co(NH_3)_5(H_2O)]^{3+} < [Co(NH_3)_6]^{3+}$

(D) $[Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-} < [Co(NH_3)_5(Cl)]^{2+} < [Co(NH_3)_5(H_2O)]^{3+}$

Answer (A)

Sol. Spectrochemical series

$(CN^-) > NH_3 > H_2O > Cl^-$

$[Co(CN)_6]^{3-} > [Co(NH_3)_6]^{3+} > [Co(NH_3)_5(H_2O)]^{3+} > [Co(NH_3)_5(Cl)]^{2+}$ (order of Δ_o)

So order of λ_{max} will be opposite.

3. One of the products formed from the reaction of permanganate ion with iodide ion in neutral aqueous medium is

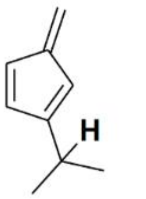
(A) I_2 (B) IO_3^-

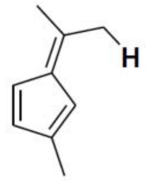
(C) IO_4^- (D) IO_2^-

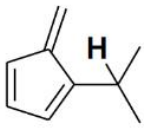
Answer (B)

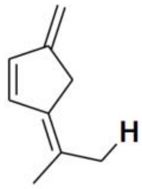
Sol. $2KMnO_4 + H_2O + KI \rightarrow 2MnO_2 + 2KOH + KIO_3$

4. Consider the depicted hydrogen (**H**) in the hydrocarbons given below. The most acidic hydrogen (**H**) is

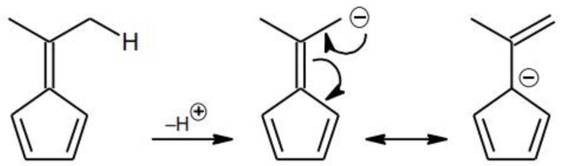
(A) 

(B) 

(C) 

(D) 

Answer (B)

Sol. 

The conjugate base of (B) formed is most stable hence, **H** is most acidic in (B)

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

5. Regarding the molecular orbital (MO) energy levels for homonuclear diatomic molecules, the **INCORRECT** statement(s) is(are)

(A) Bond order of Ne_2 is zero.

(B) The highest occupied molecular orbital (HOMO) of F_2 is σ-type.

(C) Bond energy of O_2^+ is smaller than the bond energy of O_2.

(D) Bond length of Li_2 is larger than the bond length of B_2.

Answer (B, C)

Sol. (A) According to MOT, number of Bonding electrons and number of antibonding electron in Ne_2 is same

So, $BO = \frac{N_b - N_a}{2}$

is zero

(A) is correct

(B) $F_2 \Rightarrow \sigma_{1s}^2 \sigma_{1s}^{*2} \sigma_{2s}^2 \sigma_{2s}^{*2} \sigma_{2p_z}^2 \pi_{2p_x}^2 = \pi_{2p_y}^2$

$\pi_{2p_x}^{*2} = \pi_{2p_y}^{*2} \sigma_{2p_z}^{*}$

HOMO of F_2 is π-type not σ-type

(B) is incorrect

(C) $BE \propto BO$

BO of O_2 is = 2

BO of O_2^+ is = 2.5

BE of O_2^+ > BE of O_2

(C) is incorrect

(D) Bond length of B_2 = 118 pm

Bond length of Li_2 = 267 pm

B.l. of Li_2 > B.l. of B_2

(D) is correct

6. The pair(s) of diamagnetic ions is(are)

(A) La^{3+}, Ce^{4+}

(B) Yb^{2+}, Lu^{3+}

(C) La^{2+}, Ce^{3+}

(D) Yb^{3+}, Lu^{2+}

Answer (A, B)

Sol. (A) $\left.\begin{array}{l} La^{3+} \Rightarrow [Xe]4f^0 \\ Ce^{4+} \Rightarrow [Xe]4f^0 \end{array}\right\}$ diamagnetic

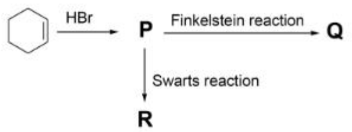
(B) $\left.\begin{array}{l} Yb^{2+} \Rightarrow [Xe]4f^{14} \\ Lu^{3+} \Rightarrow [Xe]4f^{14} \end{array}\right\}$ diamagnetic

(C) $La^{2+} \Rightarrow [Xe]5d^1$, $Ce^{3+} \Rightarrow [Xe]4f^1$ } Paramagnetic

(D) $Yb^{3+} \Rightarrow [Xe]4f^{13}$, $Lu^{2+} \Rightarrow [Xe]4f^{14}5d^1$ } Paramagnetic

Diamagnetic species are those which have all electrons paired

7. For the reaction sequence given below, the correct statement(s) is(are)



(In the options, X is any atom other than carbon and hydrogen, and it is different in P, Q and R)

(A) C–X bond length in **P**, **Q** and **R** follows the order **Q** > **R** > **P**.

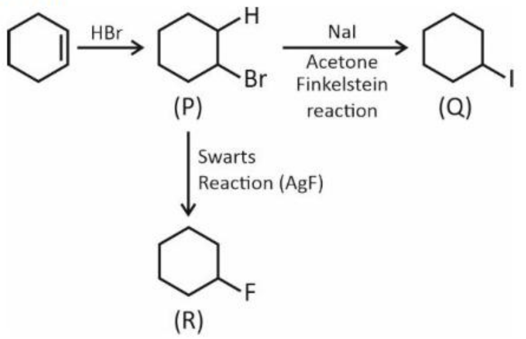
(B) C–X bond enthalpy in **P**, **Q** and **R** follows the order **R** > **P** > **Q**.

(C) Relative reactivity toward S_N2 reaction in **P**, **Q** and **R** follows the order **P** > **R** > **Q**.

(D) pK_a value of the conjugate acids of the leaving groups in **P**, **Q** and **R** follows the order **R** > **Q** > **P**.

Answer (B)

Sol.



(A) Bond length order $C-F < C-Br < C-I$

$R < P < Q$

(B) Bond enthalpy $\Rightarrow R > P > Q$

(C) Reactivity towards S_N2 $C-I > C-Br > C-F$

$Q > P > R$

(D)

	P	Q	R
Leaving group	$Br^{\ominus}$	$I^{\ominus}$	$F^{\ominus}$
Conjugate acid	HBr	HI	HF

pK_a order $HI < HBr < HF$

$Q < P < R$

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places**, truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

8. In an electrochemical cell, dichromate ions in aqueous acidic medium are reduced to Cr^{3+}. The current (in amperes) that flows through the cell for 48.25 minutes to produce 1 mole of Cr^{3+} is _______.

Use: 1 Faraday = 96500 C mol^{-1}

Answer (100.00)

Sol. $Cr_2O_7^{2-} + H^+ \longrightarrow Cr^{3+}$

g eq of Cr^{3+} produced = Faraday of charge passed

$$\text{Number of Moles} \times \text{n-factor} = \frac{i \times t}{96500}$$

$$1 \times 3 = \frac{i \times 2895}{96500}$$

i = 100 A

9. At 25 °C, the concentration of H^+ ions in 1.00×10^{-3} M aqueous solution of a weak monobasic acid having acid dissociation constant (K_a) of 4.00×10^{-11} is $X \times 10^{-7}$ M. The value of X is _______.

Use: Ionic product of water (K_w) = 1.00×10^{-14} at 25 °C

Answer (02.24)

Sol. $\underset{c(1-\alpha)}{HA} \rightleftharpoons \underset{c\alpha}{H^+} + \underset{c\alpha}{A^-}$

$$K_a = \frac{c\alpha^2}{1-\alpha}$$

$$H^+_{total} = \sqrt{K_{a_1c_1} + K_{a_2c_2} + K_w}$$

$$H^{+}_{total} = \sqrt{(4\times10^{-11}\times10^{-3})+1\times10^{-14}}$$

$$= \sqrt{4\times10^{-14}+10^{-14}}$$

$$= \sqrt{5\times10^{-14}}$$

$$= \sqrt{5}\times10^{-7}$$

$$= 2.236\times10^{-7}$$

X = 2.24

10. Molar volume (V_m) of a van der Waals gas can be calculated by expressing the van der Waals equation as a cubic equation with V_m as the variable. The ratio (in mol dm^{-3}) of the coefficient of V_m^2 to the coefficient of V_m for a gas having van der Waals constants a = 6.0 dm^6 atm mol^{-2} and b = 0.060 dm^3 mol^{-1} at 300 K and 300 atm is ______.

Use: Universal gas constant (R) = 0.082 dm^3 atm mol^{-1} K^{-1}

Answer (–07.10)

Sol. $$\left(P+\frac{a}{V_m^2}\right)(V_m-b)=RT$$

$$PV_m^3-PbV_m^2-RTV_m^2+aV_m-ab=0$$

$$V_m^3-\left(b+\frac{RT}{P}\right)V_m^2+\frac{aV_m}{P}-\frac{ab}{P}=0$$

$$\frac{\text{Coefficient of } V_m^2}{\text{Coefficient of } V_m}=\frac{-\left(b+\frac{RT}{P}\right)}{+\frac{a}{P}}$$

$$=-\left(\frac{bP+RT}{a}\right)$$

$$=-\left(\frac{0.06\times300+0.082\times300}{a}\right)$$

$$=-\left(\frac{18+24.6}{6}\right)=-\frac{42.6}{6}=-7.1$$

Answer : –7.10

11. Considering ideal gas behavior, the expansion work done (in kJ) when 144 g of water is electrolyzed completely under constant pressure at 300 K is _______.

Use: Universal gas constant (R) = 8.3 J K^{-1} mol^{-1}; Atomic mass (in amu): H = 1, O = 16

Answer (29.88)

Sol. Number of moles of $H_2O = \frac{144}{18} = 8$ moles

$2H_2O \rightarrow 2H_2 + O_2$

2 moles water $\rightarrow$ 3 moles of gases

8 moles water $\rightarrow$ 12 moles of gases

Volume of gases $= \frac{12 \times R \times 300}{P}$ m^3

Work done = – P_{ext} ΔV

$$= -P_{ext} \times \left[\frac{12 \times 8.3 \times 300}{P} - 0\right]$$

$= -29880$ J

$= -29.88$ kJ

Ans. 29.88

12. The monomer **(X)** involved in the synthesis of Nylon 6,6 gives positive carbylamine test. If 10 moles of **X** are analyzed using Dumas method, the amount (in grams) of nitrogen gas evolved is _______.

Use: Atomic mass of N (in amu) = 14

Answer (280.00)

Sol. Monomer of Nylon 6,6 $\Rightarrow$ Adipic acid

+

hexamethylene diamine

$\downarrow$

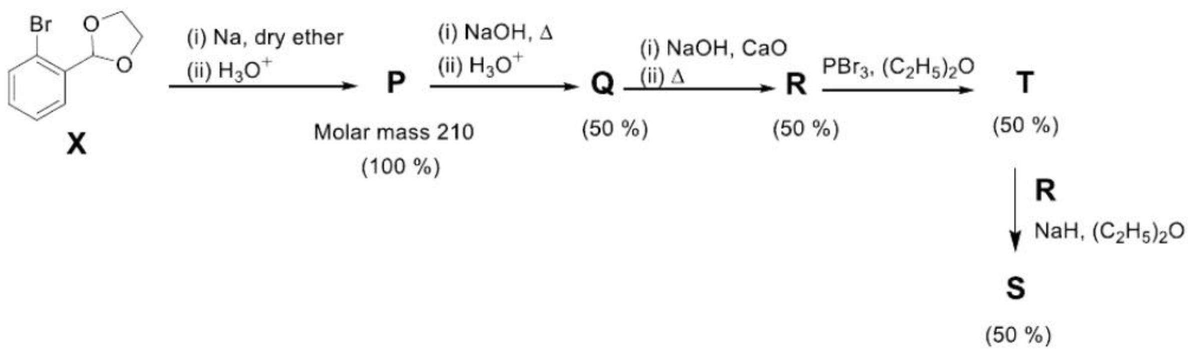
+ve carbylamine test

$$H_6N(CH_2)_6NH_2 \xrightarrow{\text{Dumas}} N_2$$

1 mole	1 mole
10 mole	10 mole

Mass of N_2 formed = 10 × 28 = 280 g

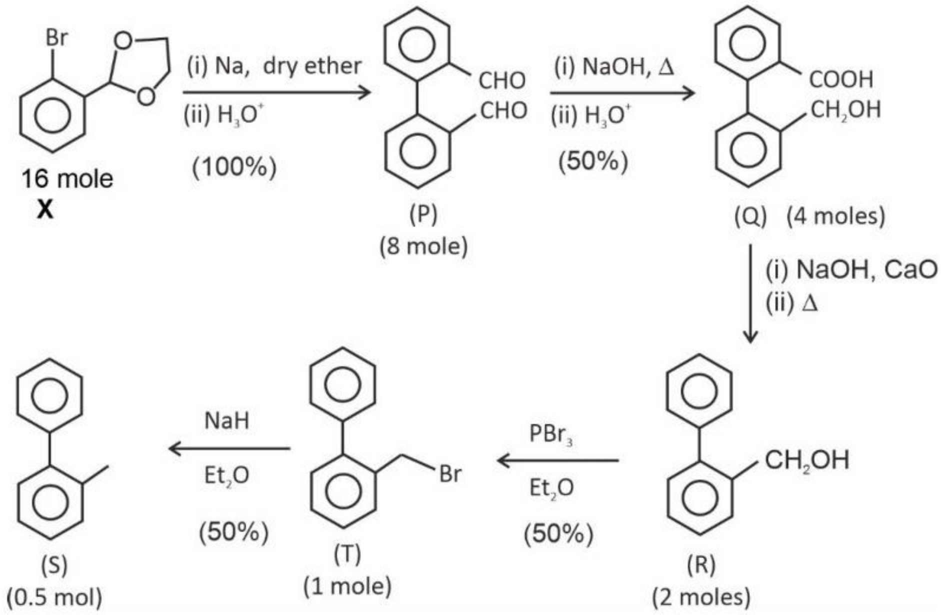
13. The reaction sequence given below is carried out with 16 moles of **X**. The yield of the major product in each step is given below the product in parentheses. The amount (in grams) of S produced is______.



Use: Atomic mass (in amu): H = 1, C = 12, O = 16, Br = 80

Answer (84.00)

Sol.



∴ Mass of 'S' is = $\frac{1}{2} \times 168 = 84$ g

SECTION 4 (Maximum Marks : 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

14. The correct match of the group reagents in **List-I** for precipitating the metal ion given in **List-II** from solutions, is

	List-I		**List-II**
(P)	Passing H_2S in the presence of NH_4OH	(1)	Cu^{2+}
(Q)	$(NH_4)_2CO_3$ in the presence of NH_4OH	(2)	Al^{3+}
(R)	NH_4OH in the presence of NH_4Cl	(3)	Mn^{2+}
(S)	Passing H_2S in the presence of dilute HCl	(4)	Ba^{2+}
		(5)	Mg^{2+}

(A) $P \to 3; Q \to 4; R \to 2; S \to 1$

(B) $P \to 4; Q \to 2; R \to 3; S \to 1$

(C) $P \to 3; Q \to 4; R \to 1; S \to 5$

(D) $P \to 5; Q \to 3; R \to 2; S \to 4$

Answer (A)

Sol. Passing H_2S in the presence of NH_4OH is group reagent for Group IV cations -Mn^{2+}

NH_4OH in the presence of NH_4Cl is group reagent for Group III cations -Al^{3+}

Passing H_2S in the presence of HCl is Group reagent for group II cations -Cu^{2+}

$(NH_4)_2CO_3$ in the presence of NH_4OH is used for detection of Ba^{2+}

$P \to 3; Q \to 4; R \to 2; S \to 1$

15. The major products obtained from the reactions in **List-II** are the reactants for the named reactions mentioned in **List-I**. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

	List-I		**List-II**
(P)	Stephen reaction	(1)	Toluene $\xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CrO_2Cl_2/CS_2}$
(Q)	Sandmeyer reaction	(2)	Benzoic acid $\xrightarrow{\text{(i) } PCl_5 \text{ (ii) } NH_3 \text{ (iii) } P_4O_{10},\ \Delta}$
(R)	Hoffmann bromamide degradation reaction	(3)	Nitrobenzene $\xrightarrow{\text{(i) Fe, HCl (ii) HCl, } NaNO_2 \text{ (273-278 K), } H_2O}$
(S)	Cannizzaro reaction	(4)	Toluene $\xrightarrow{\text{(i) } Cl_2/h\nu,\ H_2O \text{ (ii) Tollen's reagent (iii) } SO_2Cl_2 \text{ (iv) } NH_3}$
		(5)	Aniline $\xrightarrow{\text{(i) } (CH_3CO)_2O\text{, Pyridine (ii) } HNO_3,\ H_2SO_4\text{, 288 K (iii) aq. NaOH}}$

(A) $P \to 2; Q \to 4; R \to 1; S \to 3$

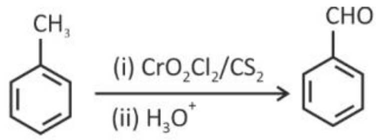
(B) $P \to 2; Q \to 3; R \to 4; S \to 1$

(C) $P \to 5; Q \to 3; R \to 4; S \to 2$

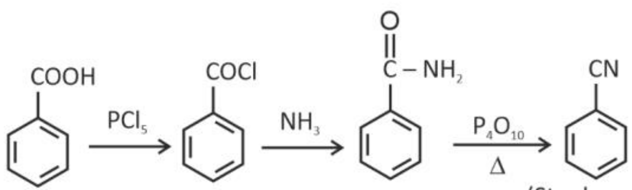
(D) $P \to 5; Q \to 4; R \to 2; S \to 1$

Answer (B)

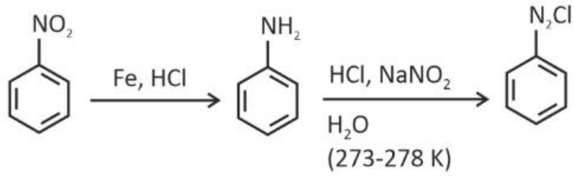
Sol.

(1) 

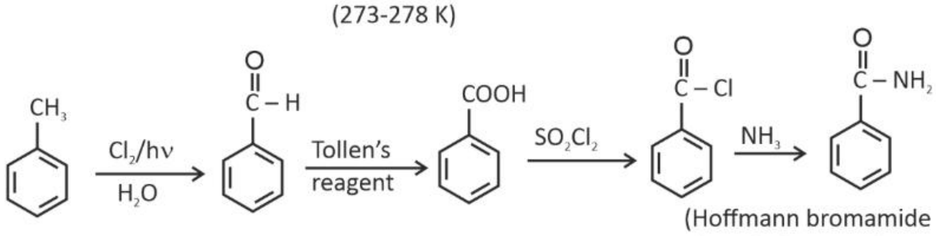
(Cannizzaro reaction reactant)

(2) 

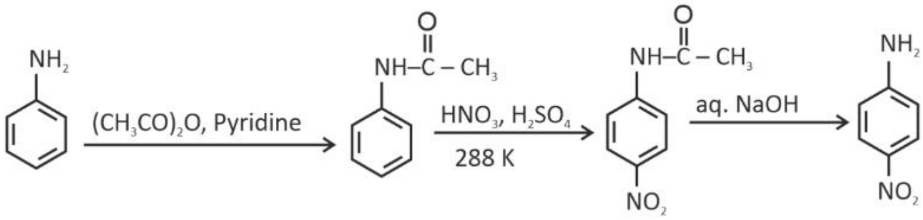
(Stephen reaction reactant)

(3) 

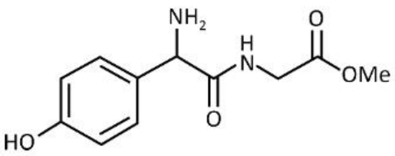
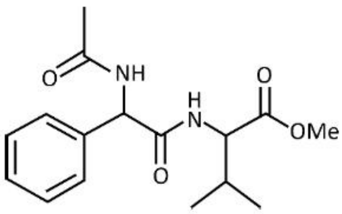
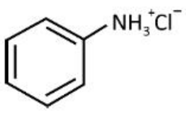
(Sandmeyer reaction reactant)

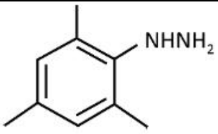
(4) 

(Hoffmann bromamide degradation reaction reactant)

(5) 

16. Match the compounds in **List-I** with the appropriate observations in **List-II** and choose the correct option.

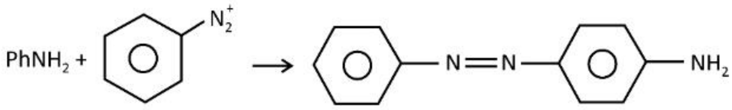
	List-I		List-II
(P)		(1)	Reaction with phenyl diazonium salt gives yellow dye.
(Q)		(2)	Reaction with ninhydrin gives purple color and it also reacts with $FeCl_3$ to give violet color.
(R)		(3)	Reaction with glucose will give corresponding hydrazone.

(S)		(4)	Lassaigne extract of the compound treated with dilute HCl followed by addition of aqueous $FeCl_3$ gives blood red color.
		(5)	After complete hydrolysis, it will give ninhydrin test and it **DOES NOT** give positive phthalein dye test.

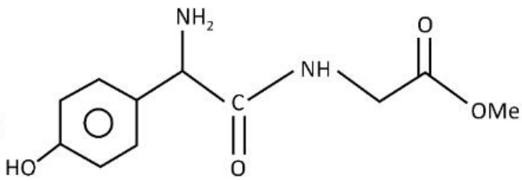
(A) P → 1; Q → 5; R → 4; S → 2
(B) P → 2; Q → 5; R → 1; S → 3
(C) P → 5; Q → 2; R → 1; S → 4
(D) P → 2; Q → 1; R → 5; S → 3

Answer (B)

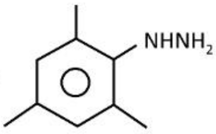
Sol.



(Yellow)

(P) 

→ Ninhydrin test
(α-amino acid with primary amino group gives purple color)

(S) 

→ Reaction with glucose will give corresponding hydrazone.